

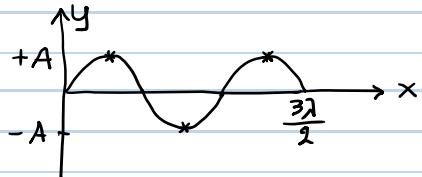
ΘΕΜΑ Α

A₁-δ, A₂-δ, A₃-α, A₄-α, A₅ ΣΣΛΛΛ

ΘΕΜΑ Β

B1-B Η θέση $x=0$ στη ΘΙ και 3 σημεία των χαρδιών είναι

$$v_{\max} \rightarrow y = \pm A$$



άρα σιάσονται κυτώρες

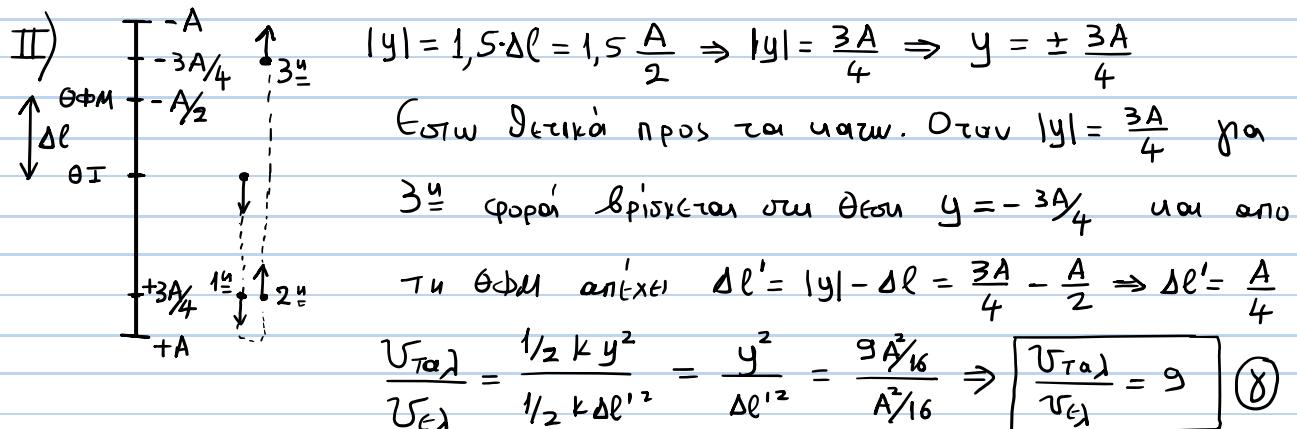
$$\text{έχει τη θέση } x_1 = \frac{3\lambda}{2}$$

$$\Delta t = t_2 - t_1 = \frac{3T}{2} \rightarrow \Delta x = v \Delta t \Rightarrow x_2 - x_1 = \frac{\lambda}{T} \frac{3T}{2} \Rightarrow x_2 - \frac{3\lambda}{2} = \frac{3\lambda}{2} \Rightarrow x_2 = 3\lambda \quad (6)$$

B2 I-α, II-γ Ι) ΘΙ $\sum F = 0 \Rightarrow F_G = mg \Rightarrow k\Delta l = mg \Rightarrow \frac{k}{m} = \frac{g}{\Delta l}$

$$U_0 = U_{\max} = \omega A \quad \text{όπου} \quad D = k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta l}}$$

$$\Rightarrow 2\sqrt{g\Delta l} = \sqrt{\frac{g}{\Delta l}} A \Rightarrow 4g\Delta l = \frac{g}{\Delta l} A^2 \Rightarrow A^2 = 4\Delta l^2 \Rightarrow A = 2\Delta l \quad (@)$$



B3-γ Τι λασπική κρούση ΑΔΟ $\vec{P}_{p1V} = \vec{P}_{p2ta} \Rightarrow P_x = P_x \Rightarrow$

$$\Rightarrow m_1 v_1 = 2m_2 v_2 \Rightarrow v_1 = 2v_2$$

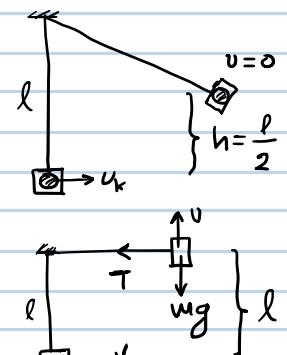
$$\text{ΘΜΚΕ } K_{T1} - K_{aex} = W_{m_2} g \Rightarrow -\frac{1}{2} m_2 v_2^2 = -m_2 g \frac{l}{2} \Rightarrow v_2 = \sqrt{gl}$$

$$\text{Άρα } v_1 = 2\sqrt{gl}$$

$$\text{Ελασπική κρούση } m_1 = m_2 \rightarrow v'_2 = v_1 = 2\sqrt{gl}$$

$$\text{ΘΜΚΕ } K_{T1} - K_{aex} = W_{mg} \Rightarrow \frac{1}{2} m_2 v^2 - \frac{1}{2} m_2 v'^2 = -m_2 g l \Rightarrow v^2 = v'^2 - 2gl = 4gl - 2gl$$

$$\Rightarrow v^2 = 2gl \rightarrow \sum F_k = m_2 a_k \Rightarrow T = m \frac{v^2}{l} = m \frac{2gl}{l} \Rightarrow T = 2mg \quad (@)$$



ΘΕΜΑ Γ

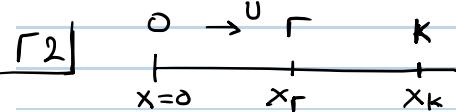
Γ1 Η πυγή έχει ψηφιστη δυνατική ενέργεια για 3^η φορά την $t = T + \frac{T}{4} = \frac{5T}{4}$

$$y = 0,4 \sin(10\pi t) \leq I \quad A = 0,4 \text{m}, \omega = 10\pi \text{ rad/s} \Rightarrow \frac{2\pi}{T} = 10\pi \Rightarrow T = 0,2 \text{ sec}$$

$$x_k = v \cdot t_k, \quad t_k = t = \frac{5T}{4} = 0,25 \text{ sec} \quad \text{όπου} \quad v = \frac{x_k}{t_k} = \frac{0,5}{0,25} \text{ m/s} \Rightarrow v = 2 \text{ m/s}.$$

$$\text{Ισχυει} \quad v = \lambda / T \Rightarrow \lambda = v \cdot T \Rightarrow \lambda = 0,4 \text{ m}$$

$$y = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi \cdot x}{\lambda}\right) \Rightarrow \boxed{y = 0,4 \sin\left(10\pi t - 5\pi x\right) \leq I}$$



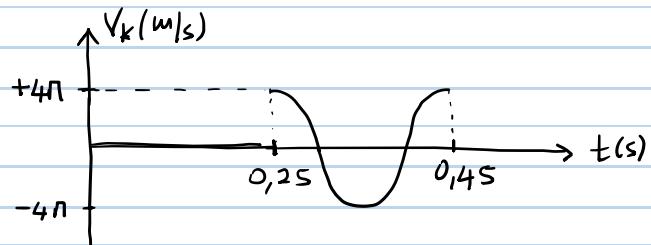
$$\phi_r - \phi_k = \frac{3\pi}{2} \text{ rad} \Rightarrow \frac{2\pi t}{T} - \frac{2\pi x_r}{\lambda} - \frac{2\pi t}{T} + \frac{2\pi x_k}{\lambda} = \frac{3\pi}{2} \Rightarrow \frac{2\pi(x_k - x_r)}{\lambda} = \frac{3\pi}{2}$$

$$x_k - x_r = \frac{3\lambda}{4} \Rightarrow 0,5 \text{m} - x_r = 0,3 \text{m} \Rightarrow \boxed{x_r = 0,2 \text{m}}$$

Γ3 $v_k = v_{\max} \sin\left(\frac{2\pi t}{T} - \frac{2\pi x_k}{\lambda}\right)$

$$\text{οπου} \quad v_{\max} = \omega A = 4\pi \text{ m/s}$$

$$\Rightarrow \boxed{v_k = 4\pi \sin\left(10\pi t - 2,5\pi\right) \leq I}$$



για $t \geq 0,25 \text{ sec}$

Γ4 $y = f(x) \quad t_1 = 0,45 \text{ sec} \rightarrow y = 0,4 \sin(10\pi \cdot 0,45 - 5\pi \cdot x)$

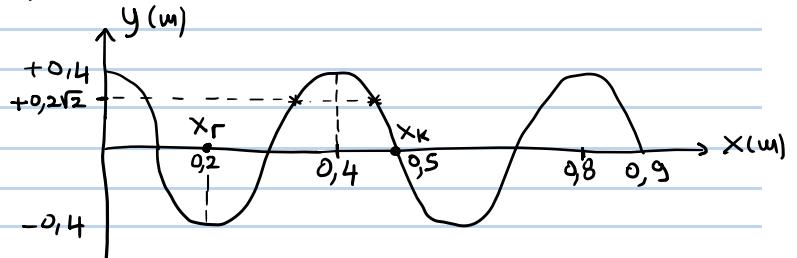
$$\boxed{y = 0,4 \sin(4,5\pi - 5\pi \cdot x) \leq I}$$

Η διανομή $x=0$: $y = 0,4 \sin(4,5\pi) = +0,4 \text{m}$

Η διανομή $x = \frac{\lambda}{4} = 0,1 \text{m}$: $y = 0,4 \sin(4\pi) = 0$

To κύτωσης έχει σεριάλει σημείο διανομής:

$$x_{TE} = v \cdot t_1 = 0,9 \text{m} \rightarrow x_{TE} = 9 \frac{\lambda}{4}$$



Γ5 $\alpha = -\alpha_{\max} \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) = -\omega^2 A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) \Rightarrow \alpha = -\omega^2 \cdot y$

$$\text{Όπως} \quad \alpha = -20\sqrt{2}\pi^2 \text{ m/s}^2 \Rightarrow -\omega^2 \cdot y = -20\sqrt{2}\pi^2 \Rightarrow -100\pi^2 \cdot y = -20\sqrt{2}\pi^2$$

$\Rightarrow y = +0,2\sqrt{2} \text{ m} = +\frac{\sqrt{2}}{2} A$. Απα στη βιτούψη σημείωση έχουν

απορριφθεί $y = +0,2\sqrt{2} \text{ m}$. (Όπως σημείωσαν από τη σημείωση στην 2)

Λύση τριγωνομετρίας: $y = +0,2\sqrt{2} \text{ m}$ τυπ $t_1 = 0,45 \text{ sec}$

$$\Rightarrow 0,4 \sin(4,5\pi - 5\pi x) = +0,2\sqrt{2} \Rightarrow \sin(4,5\pi - 5\pi x) = \frac{\sqrt{2}}{2} = \sin \frac{\pi}{4}$$

$$4,5\pi - 5\pi x = 2k\pi + \frac{\pi}{4} \Rightarrow 5x = 4,5 - \frac{1}{4} - 2k \Rightarrow x = \frac{17-8k}{20} \text{ σι } ①$$

$$4,5\pi - 5\pi x = 2k\pi + \frac{3\pi}{4} \Rightarrow 5x = 4,5 - \frac{3}{4} - 2k \Rightarrow x = \frac{15-8k}{20} \text{ σι } ②$$

Τα 3 πιούμενα σημεία βρίσκονται $x_1 < x < x_2 \Rightarrow 0,2 \text{ m} < x < 0,5 \text{ m}$

$$\Rightarrow \frac{2}{10} \text{ m} < x < \frac{5}{10} \text{ m} \Rightarrow \frac{4}{20} \text{ m} < x < \frac{10}{20} \text{ m}$$

$$① \rightarrow \frac{4}{20} < \frac{17-8k}{20} < \frac{10}{20} \Rightarrow 4 < 17-8k < 10 \Rightarrow -13 < -8k < -7$$

$$\Rightarrow \frac{7}{8} < k < \frac{13}{8} \rightarrow k=1 \rightarrow x = \frac{9}{20} \text{ m} = 0,45 \text{ m}$$

$$② \rightarrow \frac{4}{20} < \frac{15-8k}{20} < \frac{10}{20} \Rightarrow 4 < 15-8k < 10 \Rightarrow -11 < -8k < -5$$

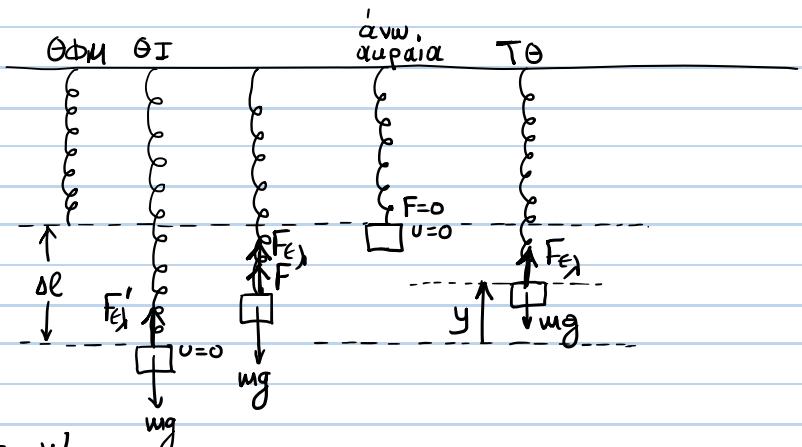
$$\Rightarrow \frac{5}{8} < k < \frac{11}{8} \rightarrow k=1 \rightarrow x = \frac{7}{20} \text{ m} = 0,35 \text{ m}$$

Αριθμητικά στα σημεία Γ και Κ υπάρχουν δύο σημεία με

επηρόχυνση $\alpha = -20\sqrt{2} \pi^2 \text{ m/s}^2$ που βρίσκονται στις θέσεις $x=0,35 \text{ m}$ και $x=0,45 \text{ m}$

ΘΕΜΑ Δ

$$m = 2 \text{ kg}, k = 50 \text{ N/m}$$



$$\Delta 1] \sum_{\text{TU}} \Theta I : \sum F = 0$$

$$\Rightarrow F_{\epsilon}' = mg \Rightarrow \Delta l = \frac{mg}{k}$$

$$\Rightarrow \Delta l = \frac{mg}{k} = 0,4 \text{ m}$$

$$\text{ΕΜΚΕ } K_{\Theta\theta} - K_{\Theta I} = W_{mg} + W_{F_{\epsilon}} + W_F$$

$$\Rightarrow 0 - 0 = -mg\Delta l + W_{F_{\epsilon}}_{\Theta I} - W_{F_{\epsilon}}_{\Theta\theta} + F\Delta l$$

$$\Rightarrow 0 = -mg\Delta l + \frac{1}{2}k\Delta l^2 + F\Delta l \Rightarrow 0 = -8 + \frac{1}{2}50\frac{16}{100} + 0,4F \Rightarrow F = 10 \text{ N}$$

$$\Delta 2] \sum_{\text{TUV}} \text{Tuxaria θεση } T\cdot\theta : \sum F = F_{\epsilon} - mg = k(\Delta l - y) - mg$$

$$\Rightarrow \sum F = k\Delta l - mg - ky \Rightarrow \sum F = -ky \rightarrow D = k$$

$$\Delta 3] D = k = m\omega^2 \Rightarrow \omega = \sqrt{k/m} = 5 \text{ rad/s} \quad A = \Delta l = 0,4 \text{ m}, v_{max} = \omega A = 2 \text{ m/s}$$

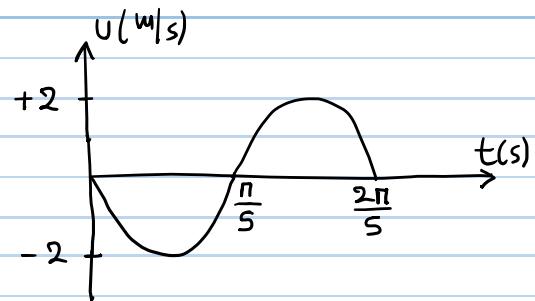
$$U = f(t) \Rightarrow U = U_{\max} \sin(\omega t + \phi_0)$$

$$\text{Την } t=0 \text{ } y=+A \Rightarrow A \sin(\omega \cdot 0 + \phi_0) = +A$$

$$\Rightarrow \sin(\phi_0) = +1 = \sin\frac{\pi}{2} \rightarrow \phi_0 = \pi/2 \text{ rad}$$

$$\text{Άρα } U = 2 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ SI}$$

$$T = \frac{2\pi}{\omega} = 0,4\pi \text{ sec} = \frac{2\pi}{5} \text{ sec}$$



$$\Delta 4 | \begin{array}{l} \text{Free body diagram: } y=0, 1\text{m}, 2\text{m} \\ \frac{dy}{dt} = F = -ky \end{array} \quad \frac{dP}{dt} = \Sigma F = -Dy = -ky$$

$$\frac{dP}{dt} = 0 \rightarrow y = 0 \text{ συ θΙ}$$

$$2 \text{ μέρος συ θΙ την } t = \frac{3T}{4} = 0,3\pi \text{ sec}$$

$$\Delta 5 | \frac{dk}{dt} = 0, |\alpha| = \frac{\alpha_{\max}}{2} \Rightarrow \omega^2 |y| = \frac{\omega^2 A}{2} \Rightarrow |y| = A/2 \rightarrow y = \pm A/2$$

$$1 \text{ μέρος } y = +A/2, v < 0, 2 \text{ μέρος } y = -A/2, v < 0$$

$$\Delta K = W_{\Sigma F} \rightarrow \frac{dk}{dt} = \frac{dW_{\Sigma F}}{dt} = \Sigma F dx = \Sigma F v = -ky \cdot v$$

$$\text{όπου } y = -A/2 = -0,2 \text{ m}$$

$$\text{και αν } \Delta ET : E = k + v \Rightarrow \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k y^2 \xrightarrow{v < 0} v = -\omega \sqrt{A^2 - y^2}$$

$$\Rightarrow v = -5 \sqrt{\frac{16}{100} - \frac{4}{100}} = -5 \sqrt{\frac{12}{100}} = -5 \frac{2\sqrt{3}}{10} \Rightarrow v = -\sqrt{3} \text{ m/s}$$

$$\Delta p \text{ or } \frac{dk}{dt} = -kyv = -50(-0,2)(-\sqrt{3}) \text{ J/s} \Rightarrow \boxed{\frac{dk}{dt} = -10\sqrt{3} \text{ J/s}}$$

$$\Delta 6 | \text{ σχετικά } \Sigma F' = m \alpha' \Rightarrow F' + mg - F_E = m \alpha' \Rightarrow 50y_{\text{θεμ}} + 4 + 20 - 50y_{\text{θεμ}} = 2\alpha'$$

$$\left(\text{όπου } F_E = ky_{\text{θεμ}} = 50 \cdot y_{\text{θεμ}} \text{ SI} \right) \Rightarrow \alpha' = 12 \text{ m/s}^2$$

Άρα για $\Delta t = \frac{\sqrt{6}}{6} \text{ sec}$ επειδή επιλέγεται ο μελίσσων πιταχνόντων κίνηση

διανούντας κατακόρυφη απόσταση $y_{\text{θεμ}} = \frac{1}{2} \alpha' \Delta t^2 = 1 \text{ m}$ και αποτά

$$\text{ζεχύτηση } v = \alpha \cdot \Delta t = 2\sqrt{6} \text{ m/s. Τοτε από τη θΙ}$$

$$\text{απέχει } |y'| = y_{\text{θεμ}} - \Delta l = 1 \text{ m} - 0,4 \text{ m} \Rightarrow |y'| = 0,6 \text{ m}$$

$$\text{ΔΕΤ τη σημερινή που καταρρίπτει } F' : E = k + v \Rightarrow$$

$$\Rightarrow \frac{1}{2} k A'^2 = \frac{1}{2} m v^2 + \frac{1}{2} k y'^2 \Rightarrow A' = \sqrt{\frac{m}{k} v^2 + y'^2} \Rightarrow \boxed{A' = 0,6\sqrt{5} \text{ m}}$$

